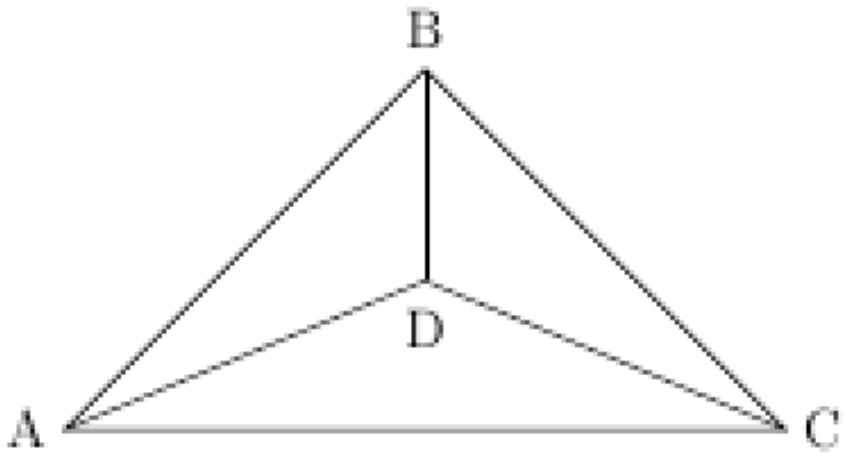
Topic: Circumscribed and inscribed circles of a triangle

Question: Complete the statement.

$\overline{AD}$, $\overline{BD}$, and $\overline{CD}$ are bisectors of $\angle BAC$, $\angle ABC$, and $\angle BCA$ respectively. Their intersection, point D, is called the incenter of the triangle.



Point D is a point that is...

Answer choices:

A Equidistant from A, B, and C

B Equidistant from the sides of ΔABC

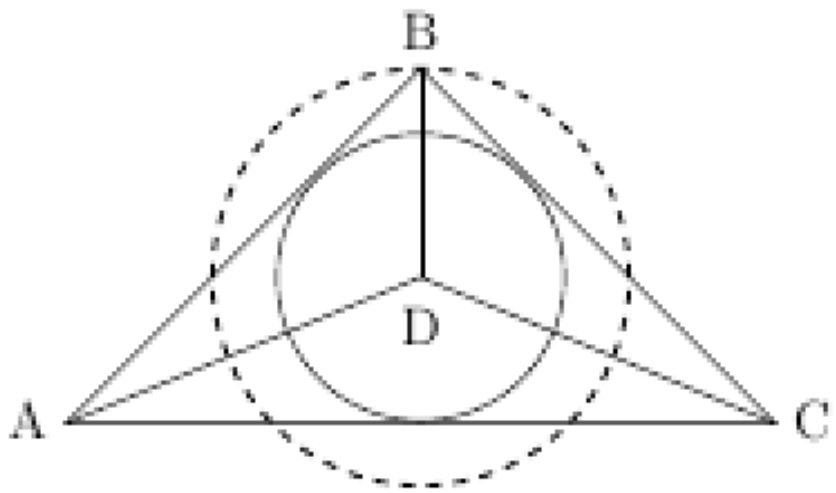
C Both A and B are correct

D Neither A nor B are correct

© Krista King Math

Solution: B

An angle bisector is at all points equidistant from the sides of the angle. The point D where the three angle bisectors meet will thus be equidistant from the sides of all angles, i.e., the three sides of the triangle. This is why you can draw an inscribed circle in the triangle using D as a center.



Notice that if you draw a bigger circle, you can't make it hit all three vertices. That tells you that D isn't equidistant from A, B, and C.

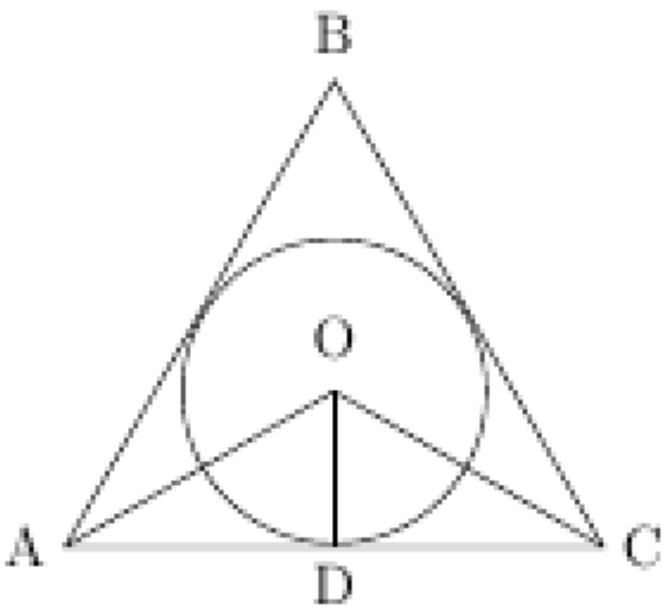
That's why the answer is not A, but B.

© Krista King Math

Topic: Circumscribed and inscribed circles of a triangle

Question: Find the radius.

In equilateral triangle ABC, with sides of 8 cm, $\overline{OA}$ and $\overline{OC}$ are bisectors of $\angle BAD$ and $\angle BCD$ respectively. $\odot O$ is inscribed in ΔABC. What is the radius, OD, of $\odot O$?



Answer choices:

A 4

B 3

C $4\sqrt{3}$

D $\frac{4\sqrt{3}}{3}$

© Krista King Math

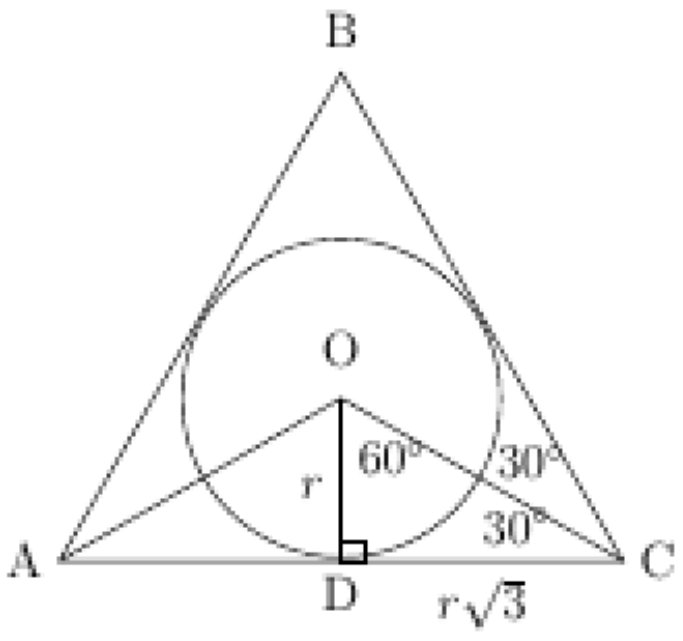
Solution: D

Let's write down what we know.

$\overline{AC}$ is tangent to the circle, so radius $\overline{OD} \perp \overline{AC}$.

$\overline{OC}$ bisects $60° \angle BCD$ into two $30°$ angles.

That leaves $60°$ for $\angle DOC$ and makes ΔDOC a 30-60-90 triangle.



In a 30-60-90 triangle if the short leg is r, the long leg is $r\sqrt{3}$.

AC is given to be 8, so DC, the long leg of ΔDOC, must be 4.

$$DC = r\sqrt{3} = 4$$

So

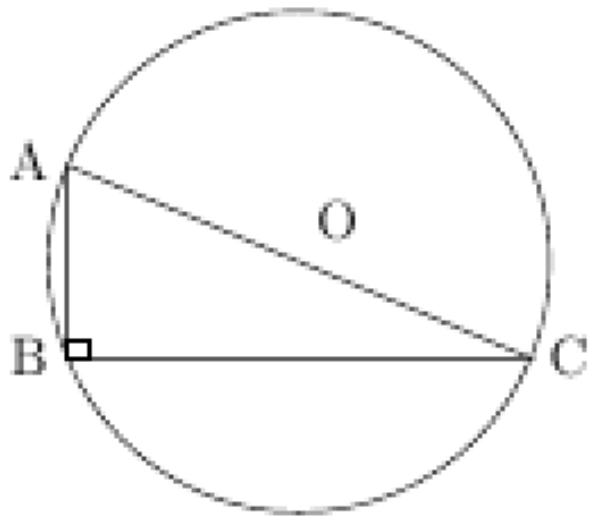
$$r = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

© Krista King Math

Topic: Circumscribed and inscribed circles of a triangle

Question: Find the radius.

In the right triangle ABC, $AB = 5$ and $BC = 12$. $\odot O$ is circumscribed around ΔABC. What is the radius of $\odot O$?



Answer choices:

A 13

B 12.5

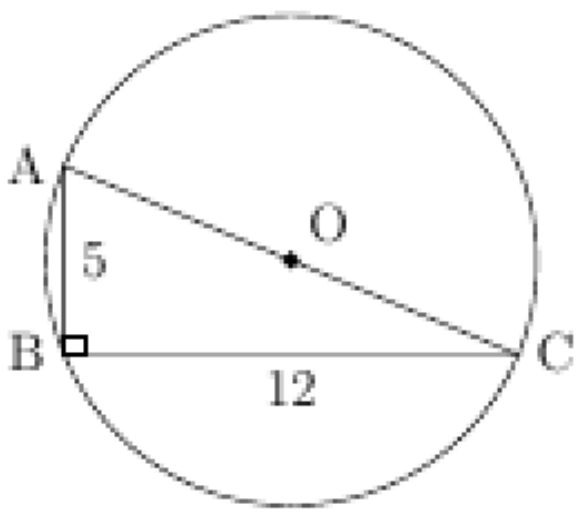
C 6.5

D 6

© Krista King Math

Solution: C

Here's what we know:



Remember, the measure of an inscribed angle is half that of the intercepted arc. So because $\angle B$ is a $90°$ inscribed angle, arc $AC = 180°$.

This guarantees that $\overline{AC}$ is a diameter, and OC is a radius.

Use $a^2 + b^2 = c^2$ to find AC.

$$5^2 + 12^2 = (AC)^2$$

$$169 = (AC)^2$$

$$AC = 13$$

The radius OC is

$$\frac{1}{2}(13) = 6.5$$

© Krista King Math